

Respostas da Lista 4

MAT 038 (GAAL)

20/6/2016

1 Respostas da Lista de Exercícios 4

1.1 Retas e Planos

<http://bit.ly/mat038-ta2> e <http://bit.ly/mat038-tb2>

Exercício 1

```
x, y, z = var('x y z')
N = vector([2, 1, -5])
P0 = (1, -2, 1)
P = (x, y, z)
P0P = vector([x-1, y+2, z-1])
```

```
N.dot_product(P0P) == 0
2*x + y - 5*z + 5 == 0
```

Exercício 2

```
x, y, z = var('x y z')
N1 = vector([1, 2, -3])
N2 = vector([2, -1, 4])
P0 = (2, 1, 0)
P0P = vector([x-2, y-1, z])
```

```
N = N1.cross_product(N2)
N.dot_product(P0P) == 0
5*x - 10*y - 5*z == 0
```

Exercício 3

```
x, y, z, t = var('x y z t')
P = (4, 1, -1)
f(t) = (2+t, 4-t, 1+2*t)
```

(a)

```
solve([f(t)[0] == 0, f(t)[1] == 0, f(t)[2] == 0], t)
[]
```

(b)

```
P0 = f(0)
```

```
P0
```

```
(2, 4, 1)
```

```
P0P = vector([4-2, 1-4, -1-1])
```

```
V = vector([1, -1, 2])
```

```
N = P0P.cross_product(V)
```

```
N.dot_product(vector([x-2, y-4, z-1])) == 0
```

```
-8*x - 6*y + z + 39 == 0
```

Exercício 4

```
x, y, z = var('x y z')
```

```
solve([x + y - z == 0, 2*x - y + 3*z == 1], x,y,z)
```

```
[[x == -2/3*r1 + 1/3, y == 5/3*r1 - 1/3, z == r1]]
```

```
P = (1, 0, -1)
```

```
P1 = (1/3, -1/3, 0)
```

```
V = vector([-2/3, 5/3, 1])
```

```
P1P = vector([1-1/3, 0+1/3, -1-0])
```

```
N = P1P.cross_product(V)
```

```
N.dot_product(vector([x-1, y-0, z+1])) == 0
```

```
2*x + 4/3*z - 2/3 == 0
```

Exercício 5

```
x, y, z = var('x y z')
```

```
solve([x - y + z + 1 == 0, x + y - z - 1 == 0], x,y,z)
```

```
[[x == 0, y == r2 + 1, z == r2]]
```

```
V = vector([0, 1, 1])
```

```
N = vector([-1, 1, -1])
```

```
V.dot_product(N)
```

```
0
```

$$P_0 = (0, 1, 0)$$

$$\begin{aligned} N \cdot \text{dot_product}(\text{vector}([x-0, y-1, z-0])) &== 0 \\ -x + y - z - 1 &== 0 \end{aligned}$$

Exercício 6

$$x, y, z, t = \text{var}('x y z t')$$

$$\begin{aligned} \text{solve}([2*x + y + z == 5, x == t, y == 2*t, z == t], x, y, z, t) \\ [[x == 1, y == 2, z == 1, t == 1]] \end{aligned}$$

$$P = (1, 2, 1)$$

Exercício 7

$$x, y, z, t, s = \text{var}('x y z t s')$$

$$\begin{aligned} \text{solve}([x == 9*t, y == 1 + 6*t, z == -2 + 3*t, x == 1 + 2*s, y == 3 + \backslash \\ s, z == 1], x, y, z, t, s) \\ [[x == 9, y == 7, z == 1, t == 1, s == 4]] \end{aligned}$$

$$P = (9, 7, 1)$$

Exercício 8

$$x, y, z = \text{var}('x, y, z')$$

(a)

$$\begin{aligned} \text{solve}([x + 2*y - 3*z - 4 == 0, x - 4*y + 2*z + 1 == 0], x, y, z) \\ [[x == 4/3*r3 + 7/3, y == 5/6*r3 + 5/6, z == r3]] \end{aligned}$$

Portanto a interseção é uma reta.

(b)

$$\begin{aligned} \text{solve}([2 - y + 4*z + 3 == 0, 4*x - 2*y + 8*z == 0], x, y, z) \\ [[x == (5/2), y == 4*r4 + 5, z == r4]] \end{aligned}$$

Portanto a interseção é uma reta.

(c)

$$\begin{aligned} \text{solve}([x - y == 0, x + z == 0], x, y, z) \\ [[x == -r5, y == -r5, z == r5]] \end{aligned}$$

Portanto a interseção é uma reta.

Exercício 9

```
x, y, z, t = var('x y z t')
P = (1, 0, 1)
N1 = vector([2, 3, 1])
N2 = vector([1, -1, 1])
```

```
N = N1.cross_product(N2)
N
(4, -1, -5)
```

```
x == 1 + t*4
y == 0 + t*(-1)
z == 1 + t*1
x == 4*t + 1
y == -t
z == t + 1
```

Exercício 10

```
x, y, z, t = var('x y z t')
x == 1 + t*1
y == 2 + t*(-1)
z == 1 + t*2
x == t + 1
y == -t + 2
z == 2*t + 1
```

Exercício 11

```
x, y, z = var('x y z')
P1 = (0, 0, 0)
V1 = vector([1, 2, -3])
P2 = (0, 1, 2)
V2 = vector([2, 4, -6])
```

```
V1.cross_product(V2)
(0, 0, 0)
```

```
P1P2 = vector([0-0, 1-0, 2-0])
P1P2
(0, 1, 2)
```

```
N = V1.cross_product(P1P2)
N
(7, -2, 1)
```

```
P1P = vector([x-0, y-0, z-0])
```

```
N.dot_product(P1P) == 0  
7*x - 2*y + z == 0
```

Exercício 12

```
N1 = vector([2, -1, 1])  
N2 = vector([1, -2, 1])
```

```
theta = arccos(abs(N1.dot_product(N2)/(N1.norm()*N2.norm())))
```

```
theta  
theta.n()  
arccos(5/6)  
0.585685543457151
```

Exercício 13

```
P1 = (0, 0, 0)  
N = vector([0-1, 1-0, 0-0])  
P0 = (1, 0, 1)  
P1P0 = vector([1-0, 0-0, 1-0])
```

```
abs(P1P0.dot_product(N)/N.norm())  
1/2*sqrt(2)
```

Exercício 14

```
t = var('t')  
A = (1, 1, 1)  
B = (0, 0, 1)
```

```
PA = vector([1-(1+t), 1-(0+t), 1-(0+t)])  
PB = vector([0-(1+t), 0-(0+t), 1-(0+t)])
```

```
PA  
PB
```

```
(-t, -t + 1, -t + 1)  
(-t - 1, -t, -t + 1)
```

```
solve(PA.norm()^2 == PB.norm()^2, t)  
[t == 0]
```

```
P = (1, 0, 0)
```

Exercício 15

$$\pi : 2x + 2y + 2z + d = 0$$

$$N = (2, 2, 2), \quad \|N\| = \sqrt{12}$$

$$P_1 = (0, 0, -d/2), \quad P_1 \in \pi$$

$$P_0 = (1, 1, 1)$$

$$\text{dist}(P_0, \pi) = \frac{|\overrightarrow{P_1 P_0} \cdot N|}{\|N\|} = \frac{|(1, 1, 1 + d/2) \cdot (2, 2, 2)|}{\sqrt{12}} = \sqrt{3}$$

$$|6 + d| = 6$$

$$d = 0 \quad \text{ou} \quad d = -12$$

Portanto

$$\pi : 2x + 2y + 2z = 0 \quad \text{ou} \quad \pi : 2x + 2y + 2z - 12 = 0$$

* Exercício 16*

Temos

$$\pi : x - y = 0,$$

$$A = (1, 1, 0),$$

$$B = (0, 1, 1).$$

Se $X = (x, y, z)$, então

$$\overrightarrow{AX} = (x - 1, y - 1, z),$$

$$\overrightarrow{BX} = (x, y - 1, z - 1).$$

Logo

$$d(A, X) = d(B, X),$$

$$X \in \pi$$

se e somente se

$$d(A, X)^2 = d(B, X)^2,$$

$$X \in \pi$$

se e somente se

$$(x - 1)^2 + (y - 1)^2 + z^2 = x^2 + (y - 1)^2 + (z - 1)^2,$$

$$x - y = 0$$

se e somente se

$$x - z = 0,$$

$$x - y = 0.$$

Portanto, os pontos do plano π que equidistam dos pontos A e B são os pontos que pertencem à reta

$$(x, y, z) = (t, t, t)$$

para $t \in \mathbb{R}$.

Exercício 17

$r \subset \pi$ se e somente se

$$(a + 2t) - 3(2 + bt) + at = 1 \quad \text{para todo } t \in \mathbb{R}.$$

Isso implica $a = 7$ e $b = 3$.

Exercício 18

```
x, y, z = var('x y z')
A = (0, 0, -1)
B = (0, 1, 0)
C = (1, 0, 1)
```

```
AB = vector([0-0, 1-0, 0-(-1)])
AC = vector([1-0, 0-0, 1-(-1)])
AX = vector([x-0, y-0, z-(-1)])
```

```
N = AB.cross_product(AC)
```

N

(2, 1, -1)

`N.dot_product(AX) == 0`

`2*x + y - z - 1 == 0`

`A0 = vector([0-0, 0-0, 0-(-1)])`

`abs(A0.dot_product(N)/N.norm())`

`1/6*sqrt(6)`