Respostas da Lista de Exercícios 4

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1 Respostas da Lista de Exercícios 4

1.1 Retas e Planos

http://bit.ly/gaal-2017

Exercício 1

```
x, y, z = var('x y z')
N = vector([2, -1, -5])
P0 = (1, -2, 1)
P = (x, y, z)
P0P = vector([x-1, y+2, z-1])
```

```
N.dot_product(P0P) == 0
```

2*x - y - 5*z + 1 == 0

Exercício 2

```
x, y, z = var('x y z')
N1 = vector([1, 2, -3])
N2 = vector([2, -1, 4])
P0 = (2, 1, 0)
P0P = vector([x-2, y-1, z])
```

```
N = N1.cross_product(N2)
N.dot_product(P0P) == 0
```

5*x - 10*y - 5*z == 0

Exercício 3

```
x, y, z, t = var('x y z t')
P = (4, 1, -1)
r(t) = (2+t, 4-t, 1+2*t)
```

(a)

```
solve([r(t)[0] == 0, r(t)[1] == 0, r(t)[2] == 0], t)
```
[]

(b)

```
P0 = r(0)
P0
```
(2, 4, 1)

```
P0P = vector([4-2, 1-4, -1-1])
V = vector([1, -1, 2])
```

```
N = P0P.cross_product(V)
```

```
N.dot_product(vector([x-2, y-4, z-1])) == 0
```
-8*x - 6*y + z + 39 == 0

Exercício 4

```
x, y, z = var('x y z')
solve([x + y - z == 0, 2*x - y + 3*z == 1], x,y,z)
```
[[x == -2/3*r2 + 1/3, y == 5/3*r2 - 1/3, z == r2]]

```
P = (1, 0, -1)
P1 = (1/3, -1/3, 0)
V = vector([-2/3, 5/3, 1])
P1P = vector([1-1/3, 0+1/3, -1-0])
```

```
N = P1P.cross_product(V)
```

```
N.dot_product(vector([x-1, y-0, z+1])) == 0
```
2*x + 4/3*z - 2/3 == 0

Exercício 5

```
x, y, z = var('x y z')
solve([x - y + z + 1 == 0, x + y - z - 1 == 0], x,y,z)
```
[[x == 0, y == r3 + 1, z == r3]]

```
V = vector([0, 1, 1])
N = vector([-1, 1, -1])
```

```
V.dot_product(N)
```
0

```
P0 = (0, 1, 0)
```

```
N.dot_product(vector([x-0, y-1, z-0])) == 0
```
-x + y - z - 1 == 0

Exercício 6

```
x, y, z, t = var('x y z t')
```

```
solve([2*x + y + z == 5, x == t, y == 2*t, z == t], x,y,z,t)
```
[[x == 1, y == 2, z == 1, t == 1]]

```
P = (1, 2, 1)
```

Exercício 7

```
x, y, z, t, s = var('x y z t s')
```

```
solve([x == 9*t, y == 1 + 6*t, z == -2 + 3*t, x == 1 + 2*s, y == 3 +\
    s, z == 1], x,y,z,t,s)
```
[[x == 9, y == 7, z == 1, t == 1, s == 4]]

```
P = (9, 7, 1)
```

Exercício 8

```
x, y, z = var('x, y, z')
```

(a)

```
solve([x + 2*y - 3*z - 4 == 0, x - 4*y + 2*z + 1 == 0], x,y,z)
```
[[x == 4/3*r4 + 7/3, y == 5/6*r4 + 5/6, z == r4]]

Portanto a interseção é uma reta.

(b)

```
solve([2 - y + 4*z + 3 == 0, 4*x - 2*y + 8*z == 0], x,y,z)
```
[[x == (5/2), y == 4*r5 + 5, z == r5]]

Portanto a interseção é uma reta.

(c)

```
solve([x - y == 0, x + z == 0], x,y,z)
```
[[x == -r6, y == -r6, z == r6]]

Portanto a interseção é uma reta.

Exercício 9

```
x, y, z, t = var('x y z t')
```

```
P = (1, 0, 1)
N1 = vector([2, 3, 1])
N2 = vector([1, -1, 1])
```

```
N = N1.cross_product(N2)
N
```

(4, -1, -5)

```
x == 1 + t*4
y == 0 + t*(-1)
z == 1 + t*1
```

x == 4*t + 1
y == -t
z == t + 1

Exercício 10

```
x, y, z, t = var('x y z t')
x == 1 + t*1
y == 2 + t*(-1)
z == 1 + t*2
```

x == t + 1
y == -t + 2
z == 2*t + 1

Exercício 11

```
x, y, z = var('x y z')
P1 = (0, 0, 0)
V1 = vector([1, 2, -3])
P2 = (0, 1, 2)
V2 = vector([2, 4, -6])
```

```
V1.cross_product(V2)
```

(0, 0, 0)

```
P1P2 = vector([0-0, 1-0, 2-0])
P1P2
```

(0, 1, 2)

```
N = V1.cross_product(P1P2)
N
```

(7, -2, 1)

```
P1P = vector([x-0, y-0, z-0])
N.dot_product(P1P) == 0
```

7*x - 2*y + z == 0

Exercício 12

```
N1 = vector([2, -1, 1])
N2 = vector([1, -2, 1])
```

```
theta = arccos(abs(N1.dot_product(N2)/(N1.norm()*N2.norm())))
```

```
theta
theta.n()
```

arccos(5/6)
0.585685543457151

Exercício 13

```
P1 = (0, 0, 0)
N = vector([0-1, 1-0, 0-0])
P0 = (1, 0, 1)
P1P0 = vector([1-0, 0-0, 1-0])
```

```
abs(P1P0.dot_product(N)/N.norm())
```

1/2*sqrt(2)

Exercício 14

```
t = var('t')
A = (1, 1, 1)
B = (0, 0, 1)
```

```
PA = vector([1-(1+t), 1-(0+t), 1-(0+t)])
PB = vector([0-(1+t), 0-(0+t), 1-(0+t)])
PA
PB
```

(-t, -t + 1, -t + 1)
(-t - 1, -t, -t + 1)

```
solve(PA.norm()^2 == PB.norm()^2, t)
```

[abs(t + 1) == -abs(t - 1), abs(t + 1) == abs(t - 1)]

```
P = (1, 0, 0)
```

Exercício 15

$$\pi : 2x + 2y + 2z + d = 0$$
$$N = (2, 2, 2), \qquad \|N\| = \sqrt{12}$$
$$P_1 = (0, 0, -d/2), \qquad P_1 \in \pi$$
$$P_0 = (1, 1, 1)$$
$$dist(P_0, \pi) = \frac{|\overrightarrow{P_1P_0} \cdot N|}{\|N\|} = \frac{|(1, 1, 1 + d/2) \cdot (2, 2, 2)|}{\sqrt{12}} = \sqrt{3}$$
$$|6 + d| = 6$$
$$d = 0 \quad \text{ou} \qquad d = -12$$

Portanto

$$\pi : 2x + 2y + 2z = 0 \qquad \text{ou} \qquad \pi : 2x + 2y + 2z - 12 = 0$$

** Exercício 16**

Temos

$$\pi : x - y = 0,$$
$$A = (1, 1, 0),$$
$$B = (0, 1, 1).$$

Se $X = (x, y, z)$, então

$$\overrightarrow{AX} = (x - 1, y - 1, z),$$
$$\overrightarrow{BX} = (x, y - 1, z - 1).$$

Logo

$$\begin{aligned} d(A,X) &= d(B,X), \\ X &\in \pi \end{aligned}$$

se e somente se

$$\begin{aligned} d(A,X)^2 &= d(B,X)^2, \\ X &\in \pi \end{aligned}$$

se e somente se

$$\begin{aligned} (x-1)^2+(y-1)^2+z^2 &= x^2+(y-1)^2+(z-1)^2, \\ x-y &= 0 \end{aligned}$$

se e somente se

$$\begin{aligned} x-z &= 0, \\ x-y &= 0. \end{aligned}$$

Portanto, os pontos do plano π que equidistam dos pontos A e B são os pontos que pertencem à reta

$$(x,y,z) = (t,t,t)$$

para $t \in \mathbb{R}$.

Exercício 17

$r \subset \pi$ se e somente se

$$(a+2t) - 3(2+bt) + at = 1 \qquad \text{para todo } t \in \mathbb{R}.$$

Isso implica $a = 7$ e $b = 3$.

Exercício 18

```
x, y, z = var('x y z')
A = (0, 0, -1)
B = (0, 1, 0)
C = (1, 0, 1)

AB = vector([0-0, 1-0, 0-(-1)])
AC = vector([1-0, 0-0, 1-(-1)])
AX = vector([x-0, y-0, z-(-1)])

N = AB.cross_product(AC)
```

```
N
```

(2, 1, -1)

```
N.dot_product(AX) == 0
```

2*x + y - z - 1 == 0

```
AO = vector([0-0, 0-0, 0-(-1)])
```

```
abs(AO.dot_product(N)/N.norm())
```

1/6*sqrt(6)